plate of $\gamma = 0.2$, clamping will only reduce the deflection to about $\frac{1}{2}$ that of the same plate simply supported.

Finally, it was found that the torsional rigidity of the edge beams has very little effect on the deflection of the plate. This is particularly true of beams with high-flexural rigidity.

A system of equations analogous to Eqs. (17–19) may be derived for other loading conditions such as the case of a hydrostatic or uniform load. Most situations arising in practice may then be solved by superposition. In general, since the solution presented herein is the Green's function for this plate system, the general expression the plate deflection under any distributed loading is given by the expression

$$w_q = \int_0^{2\pi}\int_\gamma^1 \frac{q(r,\theta)}{P} w\rho \, d\rho \, d\theta \tag{20}$$

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Thermoelastic Stress in a Rod due to Distributed Time-Dependent Heat Sources

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Introduction

PROBLEMS on determination of thermo-elastic stress and displacement in thin rods, finite or semi-infinite, under various mechanical and thermal boundary conditions have been considered by many authors, Sneddon,[1] Das,[2] Roy Choudhuri.[3] Recently moving heat source moving with a constant velocity along a finite rod has been considered by Roy Choudhuri[4] (1971). In this Note, a simple problem of thermal stress and displacement in a thin finite rod has been considered when the heat sources continuously distributed over a finite portion of the rod vary with time according to the ramp-type function, and when one end of the rod is fixed with the other free, both the ends being kept at zero temperatures. Laplace transform has been found convenient for the solution of the problem. It is believed that this particular problem has not been, so far, considered by any author.

Formulation of the Problem: Governing Equations

We consider a thin elastic rod of length l occupying the region $D: 0 \le x \le l$. The rod is heated due to heat sources which vary with time according to the ramp-type function and are distributed continuously over the length $x_0 < x < x_1$. The ends of the rod are kept at zero temperatures. If $T = T(x, t)$ is the excess of temperature over T_0, the absolute temperature of the rod in a state of zero stress and strain, then normal stress $\sigma = \sigma(x, t)$ is connected with u and T by the relation

$$\sigma = E(\partial u/\partial x - \alpha T) \tag{1}$$

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where E is Young's modulus, α is the linear coefficient of thermal expansion and $u = u(x, t)$ is the displacement at x and at time t.

In the absence of body forces, equation of motion becomes

$$\partial\sigma/\partial x = \rho(\partial^2 u/\partial t^2) \tag{2}$$

where ρ = density of the material of the rod. Eliminating σ and u between Eqs. (1) and (2), we obtain the wave equations satisfied by displacement and stress as

$$\partial^2 u/\partial x^2 = (1/v^2)(\partial^2 u/\partial t^2) + \alpha(\partial T/\partial x) \tag{3}$$

$$\partial^2 \sigma/\partial x^2 = (1/v^2)(\partial^2 \sigma/\partial t^2) + \rho\alpha(\partial^2 T/\partial t^2) \tag{4}$$

where $v = (E/\rho)^{\frac{1}{2}}$ is the velocity of elastic wave propagation.

Initial and Boundary Conditions

We suppose that the system was at rest initially. Thus the initial conditions are

$$u = \partial u/\partial t = 0, \quad \sigma = \partial\sigma/\partial t = 0 \quad \text{for} \quad 0 \le x \le l, \quad t = 0$$

and $T = 0$ for $0 \le x \le l$, $t = 0$. Also $T = 0$ at both $x = 0, l$. Since one end is kept fixed and the other free, $u = 0$ at $x = 0$, and $\partial u/\partial x = 0$ at $x = l$. The last condition follows from Eq. (1).

Solution of the Heat-Conduction Equation

Since the heat sources are continuously distributed over $x_0 < x < x_1$ and vary with time according to the ramp-type function, heat conduction equation is

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{k}\frac{\partial T}{\partial t} = -\frac{Q(x,t)}{k} = -\frac{Q_0}{k}\{H(x - x_0) - H(x - x_1)\} f(t) \tag{5}$$

where k = thermal diffusivity, $Q = w/\rho c$ where w = quantity of heat generated by the heat sources per unit time and volume, c = specific heat and

$$f(t) = (f_0/t_0)t \quad \text{for} \quad 0 \le t \le t_0$$
$$= f_0 \quad \text{for} \quad t \ge t_0$$

$H(x)$ = Heaviside step function, f_0 = const. The boundary and initial conditions for temperature are

$$T(0, t) = T(l, t) = 0 \quad \text{and} \quad T(x, 0) = 0.$$

Introducing dimensionless quantities $\xi = x/l$, $\tau = kt/l^2$, $\Theta = T/T_0$, Eq. (5) reduces to

$$(\partial^2\theta/\partial\xi^2) - (\partial\theta/\partial\tau) = -q_0 F(\tau)\{H(\xi - \xi_0) - H(\xi - \xi_1)\} \tag{6}$$

where $q_0 = Q_0 l^2/kT_0$, $\xi_0 = x_0/l$, $\xi_1 = x_1/l$,

$$F(\tau) = \frac{f_0}{\tau_0}\tau \quad \text{for} \quad 0 \le \tau \le \tau_0 \tag{7}$$
$$= f_0 \quad \text{for} \quad \tau \ge \tau_0$$

and $\tau_0 = kt_0/l^2$. Taking Laplace transform of (6), we obtain,

$$\frac{d^2\bar\theta}{d\xi^2} - p\bar\theta = -\frac{q_0 f_0}{\tau_0 p^2}(1 - e^{-\tau_0 p})\{H(\xi - \xi_0) - H(\xi - \xi_1)\} \tag{8}$$

with

$$\bar\theta(0, p) = \bar\theta(l, p) = 0. \tag{9}$$

As a solution of (8) satisfying (9), we assume

$$\bar\theta(\xi, p) = \sum_{n=1}^{\infty} A_n \sin(n\pi\xi) \tag{10}$$

where A_n is independent of ξ. We assume

$$H(\xi - \xi_0) - H(\xi - \xi_1) = \sum_{n=1}^{\infty} B_n \sin(n\pi\xi) \tag{11}$$

From Eqs. (8, 10, and 11), we obtain

$$A_n = (q_0 f_0/\tau_0 p^2)[(1 - e^{-\tau_0 p})/(p + n^2\pi^2)]B_n$$

Hence,

$$\bar{\theta}(\xi, p) = \frac{2q_0 f_0}{\tau_0} \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi)}{n\pi} \{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\} \frac{(1 - e^{-\tau_0 p})}{p^2(p + n^2\pi^2)}$$

$$= \frac{2q_0 f_0}{\tau_0} \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi)}{n^3\pi^3} \{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}$$

$$\times (1 - e^{-\tau_0 p}) \left\{\frac{1}{p^2} - \frac{1}{n^2\pi^2}\left(\frac{1}{p} - \frac{1}{p + n^2\pi^2}\right)\right\} \tag{12}$$

Hence temperature distribution is given by

$$\theta(\xi, \tau) = \frac{2q_0 f_0}{\pi\tau_0} \sum_{n=1}^{\infty} \frac{\sin(n\pi\xi)}{n} \{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}$$

$$\times \{\phi_n(\tau) - \phi_n(\tau - \tau_0)H(\tau - \tau_0)\}$$

where

$$\phi_n(\tau) = (1/n^2\pi^2)\{\tau - (1/n^2\pi^2)(1 - e^{-n^2\pi^2\tau})\} \tag{13}$$

Solution of Displacement Equation

Displacement equation (3), in dimensionless form, becomes

$$\partial^2 U/\partial\xi^2 = (\kappa^2/v^2 l^2)(\partial^2 U/\partial\tau^2) + \alpha T_0(\partial\theta/\partial\xi) \tag{14}$$

with

$$U(\xi, 0) = 0 = [\partial U(\xi, 0)/\partial\tau] \tag{15}$$

where $U = U/l$ and $[\kappa] = \mathrm{cm}^2/\mathrm{sec}$, $[v] = \mathrm{cm/sec}$. Taking Laplace transform of Eq. (14) and using Eq. (15), we obtain

$$d^2\bar{U}/d\xi^2 - (\kappa^2 p^2/v^2 l^2)\bar{U} = \alpha T_0(\partial\bar{\theta}/d\xi)$$

From Eq. (12),

$$\frac{d^2\bar{U}}{d\xi^2} - \frac{\kappa^2 p^2}{v^2 l^2}\bar{U} = \frac{2q_0 f_0 \alpha T_0}{\tau_0} \sum_{n=1}^{\infty} \cos(n\pi\xi)$$

$$\times \{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\} \frac{(1 - e^{-\tau_0 p})}{p^2(p + n^2\pi^2)} \tag{16}$$

with the conditions

$$\bar{U} = 0 \quad \text{at} \quad \xi = 0 \quad \text{and} \quad d\bar{U}/d\xi = 0 \quad \text{at} \quad \xi = 1 \tag{17}$$

Solution of Eq. (16) satisfying (17) is

$$\bar{U}(\xi, p) = \frac{2q_0 f_0 \alpha T_0}{\tau_0} \frac{v^2 l^2}{\kappa^2} \sum_{n=1}^{\infty} \{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}(1 - e^{-\tau_0 p})$$

$$\times \frac{\cosh\{(\kappa/vl)(1 - \xi)p\}}{p^2 \cosh(\kappa p/vl)} \frac{1}{(p + n^2\pi^2)(p^2 + n^2\pi^2 v^2 l^2/\kappa^2)}$$

$$- \frac{2q_0 f_0 \alpha T_0}{\tau_0} \frac{v^2 l^2}{\kappa^2} \sum_{n=1}^{\infty} \cos(n\pi\xi)\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}$$

$$\times (1 - e^{-\tau_0 p}) \frac{1}{p^2(p + n^2\pi^2)(p^2 + n^2\pi^2 v^2 l^2/\kappa^2)}.$$

Inversion

$$L^{-1}\left[\frac{(1 - e^{-\tau_0 p})}{p^2(p + n^2\pi^2)(p^2 + n^2\pi^2 v^2 l^2/\kappa^2)}\right] = \frac{\kappa^2}{n^2\pi^2 v^2 l^2}$$

$$[\phi_n(\tau) - \phi_n(\tau - \tau_0)H(\tau - \tau_0) - \psi_n(\tau) + \psi_n(\tau - \tau_0)H(\tau - \tau_0)]$$

where

$$\psi_n(\tau) = L^{-1}\left[\frac{1}{(p + n^2\pi^2)(p^2 + n^2\pi^2 v^2 l^2/\kappa^2)}\right]$$

$$= \frac{\lambda \sin[(n\pi vl/\kappa)\tau - \phi]}{(n^2\pi^2 v^2 l^2/\kappa^2 + n^4\pi^4)} + \frac{e^{-n^2\pi^2\tau}}{(n^2\pi^2 v^2 l^2/\kappa^2 + n^4\pi^4)},$$

$$\lambda \cos\phi = n\pi\kappa/vl, \quad \lambda \sin\phi = 1, \quad \phi = \tan^{-1}(vl/n\pi\kappa)$$

Also

$$L^{-1}\left[\frac{\cosh\{(\kappa/vl)(1 - \xi)p\}}{p^2 \cosh(\kappa p/vl)}\right]$$

$$= \frac{\kappa}{vl}\left[\frac{vl}{\kappa}\tau - \frac{8}{\pi^2} \sum_{v=1}^{\infty} \frac{\sin\{[(2v - 1)/2]\pi\xi\}}{(2v - 1)^2} \sin\frac{(2v - 1)\pi vl}{2\kappa}\tau\right]$$

Hence, by the convolution theorem of Laplace transform,

$$U(\xi, \tau) = \frac{2q_0 f_0 \alpha T_0}{\tau_0} \frac{v^2 l^2}{\kappa^2} \sum_{n=1}^{\infty} \{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}$$

$$\times \{\chi_n(\xi, \tau) - \chi_n(\xi, \tau - \tau_0)H(\tau - \tau_0)\} - \frac{2q_0 f_0 \alpha T_0}{\tau_0}$$

$$\times \frac{v^2 l^2}{\kappa^2} \sum_{n=1}^{\infty} \cos(n\pi\xi)\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}$$

$$\times (\kappa^2/n^2\pi^2 v^2 l^2)\{\phi_n(\tau) - \phi_n(\tau - \tau_0)H(\tau - \tau_0) - \psi_n(\tau)$$

$$+ \psi_n(\tau - \tau_0)H(\tau - \tau_0)\} \tag{18}$$

where

$$\chi_n(\xi, \tau) = L^{-1}\left[\frac{\cosh\{(\kappa/vl)(1 - \xi)p\}}{p^2 \cosh(\kappa p/vl)} \times \frac{1}{(p + n^2\pi^2)(p^2 + n^2\pi^2 v^2 l^2/\kappa^2)}\right]$$

$$= \frac{\kappa\lambda}{vln\pi(v^2 l^2 n^2\pi^2/\kappa^2 + n^4\pi^4)}$$

$$\times \left\{\tau\cos\phi - \frac{\kappa}{n\pi vl}(\sin\langle(n\pi vl/\kappa)\tau - \phi\rangle + \sin\phi)\right\}$$

$$+ \frac{1}{n^2\pi^2(n^4\pi^4 + n^2\pi^2 v^2 l^2/\kappa^2)}\left\{\tau - \frac{1}{n^2\pi^2}(1 - e^{-n^2\pi^2\tau})\right\}$$

$$- \frac{8}{n^2\pi^2} \sum_{v=1}^{\infty} \frac{\sin\{[(2v - 1)/2]\pi\xi\}}{(2v - 1)^2} \frac{\kappa}{vl}$$

$$\times \frac{1}{(n^2\pi^2 + v^2 l^2/\kappa^2)(n^4\pi^4 + \langle 2v - 1\rangle^2\pi^2 v^2 l^2/4\kappa^2)^{\frac{1}{2}}}$$

$$\times \left\{\sin\left(\frac{\langle 2v - 1\rangle\pi vl\tau}{2\kappa} - \tan^{-1}\frac{\langle 2v - 1\rangle vl}{2\kappa n^2\pi}\right)\right.$$

$$\left.+ e^{-n^2\pi^2\tau}\sin\left(\tan^{-1}\frac{\langle 2v - 1\rangle vl}{2\kappa n^2\pi}\right)\right\} - \frac{4\lambda\kappa}{vl\pi^3}$$

$$\times \frac{1}{(n^4\pi^4 + n^2\pi^2 v^2 l^2/\kappa^2)} \sum_{v=1}^{\infty} \frac{\sin\{[(2v - 1)/2]\pi\xi\}}{(2v - 1)^2}$$

$$\times \left\{\frac{\sin[(n\pi vl/\kappa)\tau - \phi] + \sin[\phi + (\langle 2v - 1\rangle\pi vl/2\kappa)\tau]}{n + (2v - 1)/2}\right.$$

$$\left.- \frac{\sin[(n\pi vl/\kappa)\tau - \phi] + \sin[\phi - (\langle 2v - 1\rangle\pi vl/2\kappa)\tau]}{n - (2v - 1)/2}\right\} \tag{19}$$

From Eq. (18) it is evident that $(\partial U/\partial\xi) = 0$ for $\xi = 1$. Also, it can be shown that $\chi_n(0, \tau) = (\kappa^2/n^2\pi^2 v^2 l^2)\{\phi_n(\tau) - \psi_n(\tau)\}$. Hence, $U(0, \tau) = 0$. Also $U(\xi, 0) = \partial U(\xi, 0)/\partial\tau = 0, 0 \leq \xi \leq 1$. Thus, the solution satisfies the boundary and initial conditions. The normal stress given by

$$\frac{\sigma(\xi,\tau)}{E} = \frac{\partial U}{\partial \xi} - \alpha T_0 \theta(\xi,\tau) = \frac{2q_0 f_0 \alpha T_0}{\tau_0}\frac{v^2 l^2}{\kappa^2}\sum_{n=1}^{\infty} \times \cos(n\pi\xi_0) - \cos(n\pi\xi_1) \times \left\{\frac{\partial \chi_n}{\partial \xi} - \frac{\partial \chi_n(\xi, \tau-\tau_0)}{\partial \xi} H(\tau-\tau_0)\right\}$$

$$- \frac{2q_0 f_0 \alpha T_0}{\pi \tau_0}\sum_{n=1}^{\infty} \times \frac{\sin(n\pi\xi)}{n}\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\} \times \{\psi_n(\tau) - \psi_n(\tau-\tau_0)H(\tau-\tau_0)\}.$$

By Eq. (19), $\sigma = 0$ at $\xi = 1$. Hence, the condition of the end $\xi = 1$ being stress-free is satisfied. From the expressions of U and σ for $\tau > \tau_0$, it is evident that both U, σ remain bounded as $\tau \to \infty$, which is expected.

Alternate Solution to Heat Conduction Equation

For point impulsive heat source, we have

$$(\partial^2\theta/\partial\xi^2) - (\partial\theta/\partial\tau) = -q_0\,\delta(\tau)\delta(\xi-\xi'), \quad \xi_0 < \xi' < \xi_1.$$

Taking Laplace transform,

$$(d^2\bar\theta/d\xi^2) - p\bar\theta = -q_0\,\delta(\xi-\xi')$$

We assume

$$\bar\theta = \sum_{n=1}^{\infty} A_n \sin(n\pi\xi),$$

since

$$\bar\theta(0,p) = \bar\theta(1,p) = 0.$$

Also from

$$\delta(\xi-\xi') = \sum_1^{\infty} c_n \sin(n\pi\xi), \quad c_n = 2\sin(n\pi\xi')$$

Hence substituting, we have

$$A_n = \{[2q_0 \sin(n\pi^{T'})]/(p+n^2\pi^2)\}, \quad n = 1,2,3,\ldots\infty$$

Therefore,

$$\bar\theta(\xi,p) = 2q_0\sum_{n=1}^{\infty}\frac{\sin(n\pi\xi')\sin(n\pi\xi)}{(p+n^2\pi^2)}.$$

Hence

$$\theta(\xi,\tau) = 2q_0\sum_{n=1}^{\infty}\sin(n\pi\xi')\sin(n\pi\xi)e^{-n^2\pi^2\tau}$$

In the present problem,

$$(\partial^2\theta/\partial\xi^2) - (\partial\theta/\partial\tau) = -q_0 F(\tau)\delta(\xi-\xi')$$

Hence

$$(d^2\bar\theta/d\xi^2) - p\bar\theta = -q_0\bar F(p)\delta(\xi-\xi')$$

Therefore

$$\bar\theta = 2q_0\sum_{n=1}^{\infty}\sin(n\pi\xi')\sin(n\pi\xi)\frac{\bar F(p)}{(p+n^2\pi^2)}$$

where $\bar F(p)$ = Laplace transform of $F(\tau)$,

$$F(\tau) = (f_0/\tau_0)\tau \quad \text{for} \quad 0 \le \tau \le \tau_0$$
$$= \tau_0 \quad \text{for} \quad \tau \ge \tau_0$$

Therefore

$$\theta(\xi,\tau) = 2q_0\sum_1^{\infty}\sin(n\pi\xi)\sin(n\pi\xi')\{e^{-n^2\pi^2\tau}F(\tau)\} \quad \begin{pmatrix}\text{convolution}\\ \text{theorem}\end{pmatrix}$$

$$= 2q_0\sum_1^{\infty}\sin(n\pi\xi)\sin(n\pi\xi')\int_0^{\tau} e^{-n^2\pi^2(\tau-\eta)}F(\eta)d\eta$$

If $\tau > \tau_0$,

$$\theta(\xi,\tau) = 2q_0\sum_{n\ 1}^{\infty}\sin(n\pi\xi)\sin(n\pi\xi')e^{-n^2\pi^2\tau}$$

$$\left\{\int_0^{\tau_0} + \int_{\tau_0}^{\tau}\right\} e^{n^2\pi^2\eta}F(\eta)d\eta$$

$$= \frac{2q_0 f_0}{\tau_0}\sum_1^{\infty}\frac{\sin(n\pi\xi)\sin(n\pi\xi')}{n^2\pi^2}$$

$$\times\left[\tau_0 - \frac{1}{n^2\pi^2}\{e^{-n^2\pi^2(\tau-\tau_0)} - e^{-n^2\pi^2\tau}\}\right]$$

This is the result obtained by using convolution theorem when the heat source is concentrated at $\xi = \xi'$. For continuously distributed heat sources distributed over $\xi_0 < \xi' < \xi_1$ we integrate the preceding result with respect to ξ' between the limits $\xi' = \xi_0$, $\xi' = \xi_1$ and get

$$\theta(\xi,\tau) = \frac{2q_0 f_0}{\tau_0\pi}\sum_1^{\infty}\frac{\sin(n\pi\xi)}{n}\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}\frac{1}{n^2\pi^2}$$

$$\times\left[\tau_0 - \frac{1}{n^2\pi^2}\{e^{-n^2\pi^2(\tau-\tau_0)} - e^{-n^2\pi^2\tau}\}\right] \quad \text{for } \tau > \tau_0$$

If $\tau < \tau_0$, $F(\eta) = (f_0/\tau_0)\eta$ and

$$\theta(\xi,\tau) = \frac{2q_0 f_0}{\tau_0}\sum_{n=1}^{\infty}\sin(n\pi\xi)\sin(n\pi\xi')e^{-n^2\pi^2\tau}\int_0^{\tau}\eta e^{n^2\pi^2\eta}\,d\eta$$

$$= \frac{2q_0 f_0}{\tau_0}\sum_{n=1}^{\infty}\frac{\sin(n\pi\xi)\sin(n\pi\xi')}{n^2\pi^2}\left[\tau - \frac{1}{n^2\pi^2}(1 - e^{-n^2\pi^2\tau})\right]$$

For distributed heat sources, we have by integration,

$$\theta(\xi,\tau) = \frac{2q_0 f_0}{\pi\tau_0}\sum_{n=1}^{\infty}\frac{\sin(n\pi\xi)}{n}\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}\phi_n(\tau)$$

where

$$\phi_n(\tau) = \frac{1}{n^2\pi^2}\left[\tau - \frac{1}{n^2\pi^2}(1 - e^{-n^2\pi^2\tau})\right] \quad \text{and} \quad \tau < \tau_0$$

Hence,

$$\theta(\xi,\tau) = \frac{2q_0 f_0}{\pi\tau_0}\sum_{n=1}^{\infty}\frac{\sin(n\pi\xi)}{n}\{\cos(n\pi\xi_0) - \cos(n\pi\xi_1)\}$$

$$\times\{\phi_n(\tau) - \phi_n(\tau-\tau_0)H(\tau-\tau_0)\}$$

where $H(\tau)$ is Heaviside function.

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